MV-ALGEBRAS AND COHERENT FRAMES INDUCED BY STABLE CLOSURE OPERATIONS

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Abstract. Frames are abstract structures that offert a framework in which important algebraic and topological properties of ideals, filters or congruences in rings, lattices or others algebras can be generalized (see [1, 2, 7, 9]). Intensive work was done by Themba Dube and B. Banashewski (see:[1, 2, 6, 7]) on the frame of radical ideals of rings. B. Banaschewsky and Dube Thembe show that the frame $RId(\mathbf{A})$ of radical ideals of a ring $\mathbf{A}$ induced a lot of properties of $\mathbf{A}$ ([1, 2, 6, 7].

In this work we show that, for each stable closure operation c a on the set $Id(\mathbf{A})$ of ideals of an MV-algebra $\mathbf{A}$, the lattice $cId(\mathbf{A})$ of c-closed ideals forms a coherent frame and the collection of tuples $(\mathbf{A}, c_A)$ wih MV-homomorphisms verfying some condition forms a category noted **CMV**, with subcategory the category of MV-algebras. We show that there are two covariant functors between the category **CMV** and the category of Coherent frame and a natural transformation betwen these functors. As consequence of the preceding results, we obtain that the stable closure operation Rad on the lattice of ideals of MV-algebras induces a functor $RId : \mathbf{MV} \to \mathbf{Chfrm}$ between the category $\mathbb{MV}$ of MV-algebras and the category of Coherent frames. The set $\mathcal{O}(Max(\mathbf{A}))$ of open sets of $max(\mathbf{A})$ (where $Max(\mathbf{A})$ is endowed with the spectral topology) forms a coherent frame isomorphic to the coherent frame $RId(\mathbf{A})$.

keywords: MV-algebra, ideal, closure operation, functor, coherent frame.

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